Instituto Superior de Economia e Gestão
UNIVERSIDADE TÉCNICA DE LISBOA
DESDE 1911

Master in Actuarial Science
Loss Reserving
22-06-2018
Time allowed: 2 hours

Solutions

Instructions:

1. This paper contains **5** questions and comprises **5** pages including the title page.
2. Enter all requested details on the cover sheet.
3. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each question on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. Mobile phones and smartphones may not be used during the examination.

You have been asked to estimate the ultimate claim cost of a portfolio of long-tailed insurance that has been running for four years only. The company has given you its premiums and paid claims.

Company Statistics

Accident year	Premium M€	Paid claims M€
2010	63	5
2011	99	11
2012	155	12
2013	178	5

A quick glance convinces you that the company's statistics alone are useless for estimating the ultimate claims, given the long-tailed nature of the business.

Fortunately you have access to seven years of industry statistics, shown below.

Industry Statistics

		Accumulated paid claims by Development year (M€)						
Accident year	Premium M€	0	1	2	3	4	5	6
2006	2 329	33	139	265	464	661	847	949
2007	2 495	39	163	319	503	709	869	
2008	2 649	60	186	368	574	813		
2009	2 674	65	193	348	514			
2010	2 584	31	129	256				
2011	2 561	42	164					
2012	2 526	30						
Sum	17 817	300	974	1 556	2 055	2 183	1 716	949

1. Estimating a payment pattern from the Industry Statistics

Solutions to a-c are collected in d.

a. Estimate year-on-year development factors δ_e^* for $e = 1, \cdots, 6$. [10 marks]

b. Assume that claims paid until development year 6 make up only 2/3 (66.7%) of ultimate claim cost. Calculate the development factor that applies to the development from development year 6 to ultimate. For simplicity you may denote it by δ_7^*.

[10 marks]

c. Transform the set of estimated development factors $\delta_1^*,\cdots,\delta_7^*$ into a payment pattern $\pi_0^*,\cdots,\pi_7^*$, where π_e^* denotes the expected proportion of ultimate claim payments, that will be made in development year *e*. [10 marks]

d. Collect the results of a-c above in this table.

	Specify formula	0	1	,,,	6	7 to ultimate
δ_e^*						
Δ_e^*						
$\pi_{\leq e}^*$						
π_e^*						

The meaning of the symbols is:

δ_e^* Year-on-year development factor,

Δ_e^* Accumulated development factor,

$\pi_{\leq e}^*$ Accumulated payment proportion (current-to-ultimate),

π_e^* Incremental payment proportion (payment year to ultimate).

Please display your results as ratios with four decimals (x.xxxx) or as percentages with two decimals (x.xx%). [5 marks]

Solution to a-d at end.

e. Estimate the ultimate claim cost of the Industry Portfolio using the Chain ladder method (CL). Display your results in the table below. [10 marks]

Accident year *j*	Premium	Paid claims	$\pi_{\leq 2012-j}^*$	Estimated claim rate (CL)	Estimated Outstanding claim cost (CL)	Estimated Ultimate claim cost (CL)
Specify formula						
2006						
...						
2012						
Total				(average)		

f. Use the Cape Cod (CC) method to calculate the average claim rate. [10 marks]

Solution to e-f at end.

2. Estimating the claim cost of the company

Now you get to the task that your actually were asked to do:
to estimate the ultimate claim cost of the company portfolio using company statistics and the industry payment pattern.

a. Estimate the ultimate claim cost of the Company Portfolio using Benktander's method. Display your results in the table below.

				Estimated claim rate			Estimated claim cost (Benktander)		
Accident year j	Premium	Paid claims	$\pi^*_{\leq 2013-j}$	Chain ladder	Average	Benktander	Outstanding	Ultimate	Ultimate claim rate
Specify formula									
2010									
...									
2013									
Total				(Cape Cod average)					

[25 marks]

Solution at end.

b. The company is convinced that its own portfolio is more profitable than the industry portfolio. Just taking the numbers at face value, and leaving aside all doubts about the paucity of the company data or the relevance of the industry's payment pattern, do your results support or contradict the company's assertion?

[10 marks]

Solution: Industry average claim rate = 0.59, company 0.51, so the results do not contradict the company's assertion.

c. A board member says to you "*I've never heard of this Ben Thunder. Every actuary I know uses the chain ladder. Why aren't you using it?*"

Explain why, in your opinion, using the Chain ladder method for the Company Portfolio would not be a very good idea. Use the actual numbers you have, to illustrate your point.

[15 marks]

Solution: The volatility of its predictions make the CL unsuitable. It should not be used before the current-to-ultimate ratios are at least, say 40-50%. In this portfolio and according to the industry statistics, that takes about 4 development years, cf. question 1 d. The table in question 2 a shows the claim rates that would result from using the CL, and they vary between 24% and 92%. The claim rates from the Benktander method are stable for the last 3 years. But the Benktander method is not as insensitive as Bornhuetter-Ferguson's method.

3. Generalised linear models (GLM) can be used to model many different structures.

   a. Describe the three components that define a GLM. [10 marks]

Covariate structure
Link function/response function
Probability distribution

   b. Using GLM, propose a joint model of two insurance portfolios. The portfolios are motor insurance in two different countries or regions. You assume that they have the same development pattern but different claim rates. The claim rates in each country are constant over time, so there is no need for an accident year effect. You want to use the total statistical information from the two portfolios to estimate the development pattern and the two claim rates. [20 marks]

Model

$$\mathrm{E}\left(X_{je}^{(k)}\right) = p_j \cdot \exp\left(\beta_e + \gamma_k\right)$$

Where *k* is the country/region.

   c. Explain the meaning of the assertion "Every claim cohort is like a different portfolio when you use the chain ladder method". [10 marks]

The model in the previous question is very similar to the model underlying the chain ladder estimates. Every country is like a cohort.

4. Bühlmann-Straub model

a. Describe briefly the assumptions of the Bühlmann-Straub model for claim amounts and explain the meaning of the parameters β, φ and λ. [20 marks]

The optimal credibility estimator of the random claim level of accident year *j* has the form $\bar{b}_j = \zeta_j \hat{b}_j + (1-\zeta_j)\beta$, where $\hat{b}_j$ is the chain ladder estimator, β is the prior mean, and ζ_j is the optimal credibility factor.

b. Specify the formula for the optimal credibility factor ζ_j. [10 marks]

c. Explain in what way the Bornhuetter-Ferguson method and the chain ladder method can be seen as limiting cases of the Bühlmann-Straub credibility method. [10 marks]

a. The candidate should mention:

- **Conditional on an unobserved risk parameter that we denote by Θ_j, the incremental payments $X_{j0}, X_{j1}, \cdots$ are stochastically independent with conditional mean $\mathrm{E}(X_{je} \mid \Theta_j) = p_j b(\Theta_j)\pi_e$ and variance $\mathrm{Var}(X_{je} \mid \Theta_j) = p_j v(\Theta_j)\pi_e$.**
- **The unobserved risk parameter Θ_j is seen as the outcome of a random variable.**
- **The $\Theta_1, \cdots, \Theta_J$ are stochastically independent and identically distributed. We denote the mean and variance of the function $b(\Theta_j)$ by $\beta = \mathrm{E}(b(\Theta_j))$ and $\lambda = \mathrm{Var}(b(\Theta_j))$.**
- **We denote the mean of the function $v(\Theta_j)$ by $\varphi = \mathrm{E}(v(\Theta_j))$.**

b. Optimal credibility factor $\zeta_j = \dfrac{\lambda p_j \pi_{\le J-j}}{\lambda p_j \pi_{\le J-j} + \varphi}$

c. Bornhuetter-Ferguson is limiting case for $\lambda \to 0$.
Chain ladder is a limiting case for $\lambda \to \infty$ or $\varphi \to 0$

5. Explain the cost-of-capital method for calculating a risk margin. [15 marks]

The student should mention (3 marks each):

- **The idea of another company taking over the liability**
- **The need for solvency capital to support the liability**
- **That solvency capital is needed until all claims are settled**
- **The notional charge for providing solvency capital every year**
- **That the charge is upfront (affects the discount factor)**

Solution for Question 1 a-d

		Development year e							
	Specify formula	0	1	2	3	4	5	6	7 to ultimate
δ_e^*	$\delta_e^* = \sum_{j=1}^{J-e} X_{j,\leq e} / \sum_{j=1}^{J-e} X_{j,\leq e-1}$		360,74 %	192,10 %	158,08 %	141,66 %	125,26 %	112,04 %	150,00 %
Δ_e^*	$\Delta_e^* = \prod_{e'=0}^{e} \delta_{e'}^*$	100,00 %	360,74 %	692,98 %	1095,44 %	1551,81 %	1943,73 %	2177,80 %	3266,71 %
$\pi_{\leq e}^*$	$\pi_{\leq e}^* = \Delta_e^* / \Delta_7^*$	3,06 %	11,04 %	21,21 %	33,53 %	47,50 %	59,50 %	66,67 %	100,00 %
π_e^*	$\pi_e^* = \pi_{\leq e}^* - \pi_{\leq e-1}^*$	3,06 %	7,98 %	10,17 %	12,32 %	13,97 %	12,00 %	7,17 %	33,33 %

Solution for Question 1e-f

Accident year j	Premium	Paid claims	$\pi^*_{\leq 2012-j}$	Estimated claim rate (CL)	Estimated Outstanding claim cost (CL)	Estimated Ultimate claim cost (CL)
Specify formula	p_j	$X_{j,\leq 2012-j}$	Table lookup	$\theta^*_j = \frac{X_{j,\leq 2012-j}}{p_j \pi^*_{\leq 2012-j}}$	$\overline{X}_{j,>2012-j} = p_j \theta^*_j \left(1 - \pi^*_{\leq 2012-j}\right)$	$X_{j,\leq 2012-j} + \overline{X}_{j,>2012-j}$
2006	2 329	949	67 %	0,61	475	1 424
2007	2 495	869	60 %	0,59	591	1 460
2008	2 649	813	48 %	0,65	898	1 711
2009	2 674	514	34 %	0,57	1 019	1 533
2010	2 584	256	21 %	0,47	951	1 207
2011	2 561	164	11 %	0,58	1 321	1 485
2012	2 526	30	3 %	0,39	950	980
Total	17 817	3 595		Average 0,59	6 205	9 800

Average by Cape Cod method:

$$\theta^* = \sum_{j=2006}^{2012} X_{j,\leq 2012-j} \Big/ \sum_{j=2006}^{2012} p_j \pi^*_{\leq 2012-j} = \sum_{j=2006}^{2012} p_j \pi^*_{\leq 2012-j} \theta^*_j \Big/ \sum_{j=2006}^{2012} p_j \pi^*_{\leq 2012-j}$$

Solution for Question 2a

				Estimated claim rate			Estimated claim cost (Benktander)		
Accident year *j*	**Premium**	**Paid claims**	$\pi^*_{\leq 2013-j}$	**Chain ladder**	**Average**	**Benktander**	**Outstanding**	**Ultimate**	**Ultimate claim rate**
Specify formula	p_j	$X_{j,\leq 2013-j}$	**Table lookup**	$\theta^*_j = \frac{X_{j,\leq 2013-j}}{p_j \pi^*_{\leq 2013-j}}$	θ^*	$\bar{\theta}_j = \pi^*_{\leq 2013-j}\theta^*_j + \pi^*_{>2013-j}\theta^*$	$\bar{X}_{j,>2013-j} = p_j \bar{\theta}_j \, \pi^*_{>2013-j}$	$X_{j,\leq 2013-j} + \bar{X}_{j,>2013-j}$	$\frac{X_{j,\leq 2013-j} + \bar{X}_{j,>2013-j}}{p_j}$
2010	**63**	**5**	**34 %**	**0,24**	**0,51**	**0,42**	**18**	**23**	**0,36**
2011	**99**	**11**	**21 %**	**0,52**	**0,51**	**0,51**	**40**	**51**	**0,52**
2012	**155**	**12**	**11 %**	**0,70**	**0,51**	**0,53**	**73**	**85**	**0,55**
2013	**178**	**5**	**3 %**	**0,92**	**0,51**	**0,52**	**90**	**95**	**0,53**
Total	**495**	**33**		**Average 0,51**			**221**	**254**	**0,51**

Average by Cape Cod method:

$$\theta^* = \sum_{j=2010}^{2013} X_{j,\leq 2013-j} \Big/ \sum_{j=2010}^{2013} p_j \pi^*_{\leq 2013-j} = \sum_{j=2010}^{2013} p_j \pi^*_{\leq 2013-j} \theta^*_j \Big/ \sum_{j=2010}^{2013} p_j \pi^*_{\leq 2013-j}$$